Formative Conversation Starters: Math

GRADE 7

nwea Professional Learning

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Formative Conversation Starters

Student understanding is more about growing than it is about getting. As educators, we might speak of some students "getting it" and other students not. This conclusion, though, is not fair to students. A student who may not seem to "get it" does understand *something*, even if that *something* may not yet have grown into a robust web of thinking. To reach students' full knowledge, we can use guided conversations: Formative Conversation Starters.

Purpose: The purpose of Formative Conversation Starters is to help teachers reveal student understanding about key ideas in mathematics and to identify their students' ways of thinking.

Audience: The intended audience is teachers, who will use these questioning strategies with students.

Application: Teachers may wish to use these conversation starters in one-on-one conferences with students or in small groups.

Formative Conversation Starters approach student knowledge by presenting a single standards-based assessment item and leveraging the item to elicit conversation through clustered questioning. The goal of this activity is not to tell students what to think, but to help teachers better uncover how students are currently thinking about mathematical concepts. The conversations provide opportunities for students to communicate how they are thinking about mathematics.

Mathematical Ideas (BINSS – Big Ideas to Nurture Standards Sense-making)

As you read through these items, you will notice that we draw attention to a few specific mathematical ideas. These ideas correspond to important ways of thinking that all students should develop and continue to refine. They include:

- **Operations:** Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

- **Place Value:** Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (*or a bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

- **Comparisons:** Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is 3 times as large as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is 4 meters more than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

- **Measurement:** Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

- **Fractions:** A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

- **Formulas:** Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r2$ means three-and-a-bit copies of the square with area $r2$).

- **Variables:** Students should have ways of thinking that enable them to distinguish between unknowns that vary and unknowns that represent some fixed value. For example, in the equation $6=3x+2$, x represents some unknown fixed value that makes the equation true. In $y=3x+2$, y and x vary with each other. In $y=mx+b$, y and x vary with each other, while m and b are typically nonvarying constants (parameters) within a problem.

- **Covarying Quantities:** Single quantities can vary, but students also need to consider situations where two quantities vary together. For example, in the equation $y=3x+2$, as the quantity x varies, the quantity y varies. It can be helpful to think about the relationship between covarying quantities in terms of how changes in x result in changes in y (e.g., as x increases by 1, y changes by . . .).

- **Proportional Relationships:** Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

- **The Equal Sign:** The equal sign works in multiple ways in mathematics, even though the symbol does not change. Students should be able to think about the symbol as being relational (e.g., 2+4=5+1) and as being operational (e.g., the output of a computation), and should be able to determine which role the symbol is playing based on the situation. Keep an eye out for when students may put together equal signs. If this happens, prompting the student with questions about what the equals sign means may help. For example, suppose a student writes: 9 x 8 = 72 = 126 – 72 = 54 (*in this case, the student might be using the equal sign to say "next, I will…"*).

In the Formative Conversation Starters, you will notice that we call out mathematical ideas and the ways of thinking associated with them when it makes sense to do so. We also group discussion prompts to focus the conversations. Some groupings target core understandings that underpin the content. Other groupings elicit flexibility of thinking or extend beyond the assessment item being discussed. All of these methods are intended to provide opportunities for the teacher to listen to students and to reflect on how students might be thinking about mathematics in the standards-based assessment item.

The progressive question-and-answer strategy can be used to elicit evidence of students' ways of thinking about a topic or concept, with the purpose of guiding instruction.

How to Conduct a Formative Conversation

1. The questions for each item were developed to help teachers elicit information about students' ways of thinking about the content in the item and about mathematical ideas. These questions are suggestions, however, and not intended to be used as a script. The conversations teachers have will vary by student. While the questions for an item are laid out in a progression, teachers should vary the order to adapt to students' responses. Teachers should also keep in mind that students' responses may point to ways of thinking that are not addressed by the provided questions. In these cases, teachers should pursue those student understandings with their own line of questioning. There are several actions teachers can take to prepare for formative conversations:

    a. Become very familiar with the task and the questions ahead of time. This ensures that teachers can select the most appropriate next question based on how the students are responding.

    b. Provide students with tools to help them answer the questions. Depending on the task, these tools might include a manipulative, drawing paper, graph paper, or individual whiteboards and markers.

   c. Have a list of questions to help further probe what students are thinking. Some examples are:
      i. Can you tell me more about that?
      ii. You look like you're really thinking about this. What are you thinking?
      iii. Can you draw me a picture/write an equation?
      iv. How did you get that answer?
      v. Is there another way that you could find that answer?
   d. Make a plan to track what students say during the conversation: record the conversation, take notes, or have an observer take notes.

2. Teachers should ask questions without judgment. Student responses should not be labeled as right or wrong, and follow-up questions should be asked regardless of whether students give a correct response. Teachers should avoid commenting on students' responses other than to ask follow-up questions or to clarify what a student has said. Other students, however, should be encouraged to agree or disagree in a small group setting.

3. One of the most important parts of the formative conversation is what comes after the conversation: how will a teacher use the information about student thinking when planning instruction. Consider these suggestions for how to act on a formative conversation:
   a. Identify the different mathematical ideas addressed in the formative conversation. Where did students make connections between the ideas? Where do the connections need to be strengthened?
   b. Identify what students already understand in order to build instruction on that understanding.
   c. Identify areas where students can deepen what they already understand.
   d. Identify ways that students are comfortable with expressing mathematical ideas, and plan how to expand their capabilities. How were students most comfortable expressing or explaining what they understand? Were they more inclined to create a graphical representation of their thinking? Did they prefer to explain verbally? Did they use equations, or did they prefer to use graphs?

4. Some Conversation Starters include extension questions. These questions are provided as a way to elicit thinking beyond the grade-level of the BINSS.

7.1 Proportional Relationships

Proportional Relationships: Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

ITEM ALIGNMENT

CCSS: 7.RP.A.2

This item focuses on proportional relationships. However, it also provides an opportunity to talk about reasoning about ratios and covarying quantities.

THE CONVERSATION STARTER

This question has two parts. Use the information to answer Part A and Part B.

A restaurant owner keeps track of the food prepared for guests. On Friday, 15 pounds of vegetables were prepared for 12 guests. On Saturday, 35 pounds of vegetables were prepared for 28 guests. On Sunday, 25 pounds of vegetables were prepared for 20 guests.

Part A

Select one choice from the set to complete the sentence.

Based on this data, the number of pounds of vegetables **[is / is not]** proportional to the number of guests.

Part B

What is the constant of proportionality, if any, for this scenario?

- A. 1.25 guests per pound of vegetables
- B. 1.25 pounds of vegetables per guest
- C. there is no constant of proportionality

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

What is this problem asking you to do?

What are the quantities in this problem?

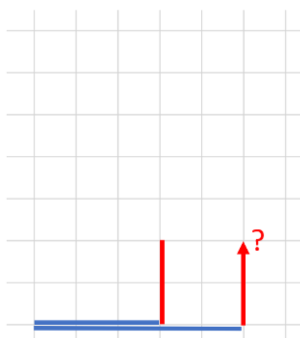
- Do the quantities vary?
- Do corresponding pounds of vegetables and number of guests seem to vary in some dependent way?
- Given the context, how should the measures vary?
- Does it look like twice the number of guests would require twice as many pounds of vegetables?

B. Content: Proportional Relationships (Meaning)

What is a proportional relationship?

C. Content: Proportional Relationships (Covarying quantities, representation)

In the image below, the length of the blue line increased as shown. If the length of the red line is proportional to the length of the blue line, what is the length of the longer red line? Can you explain it without using a formula?

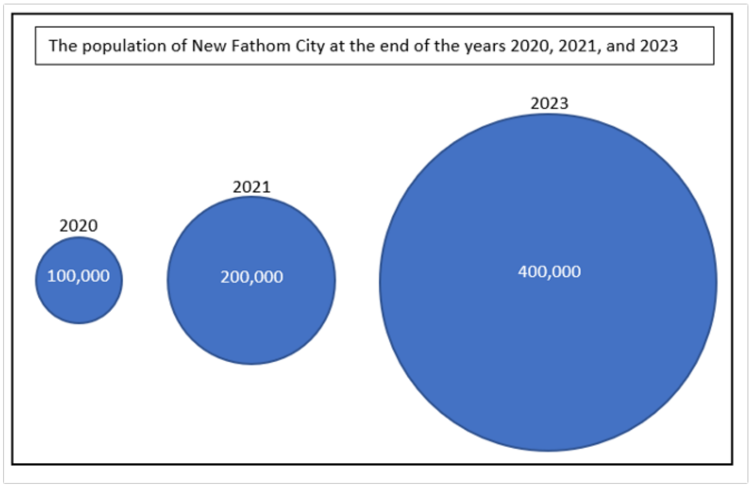


- Are there any meaningful multiplicative comparisons in the above diagram?
- What is meaningful about the number $\frac{5}{3}$ in the diagram above?
- What is meaningful about the number $\frac{2}{3}$ in the diagram above?
- If we imagine the blue line growing across all possible lengths, what happens with the red line? Are you sure?
- Can you write an equation that shows the relationship between the lengths of the blue and red lines?

D. Content: Proportional Relationships (Meaning)

In the following graphic, tell me when you see two quantities that are in proportion to each other.

- Which pairs of quantities do not seem to be in a proportional relationship?



E. Problem Solving: Strategy

Go back to the original problem about the number of pounds of vegetables a restaurant needs for a certain number of guests. How could you find the number of pounds of vegetables that would likely be prepared for 4 guests?

- Can you think of another way to find the number of pounds of vegetables that would likely be prepared for 4 guests?
- Can you figure out how many guests you could serve with 10 pounds of vegetables?
- Can you think of another way to solve the problem?

F. Content: Proportional Relationships (Representation)

Here's one way to organize the data in the original problem:

Number of Guests	G	1	12	20	28
Pounds of Vegetables			15	25	35

If the relationship between guests and pounds of vegetables is proportional, how many pounds of vegetables does one guest receive?

- Using *G* as part of your answer, what belongs in the box below *G*?
- Is there an equation that would show that relationship?
- If there are 1.25, or ⁵⁄₄, pounds of vegetables for every guest, how many guests are there per pound of vegetables?

G. Extension: Problem Solving

Suppose that in some situation, apples are proportional to bananas and bananas are proportional to cherries. Does this mean apples are proportional to cherries? Explain.

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

What is this problem asking you to do?

> ***Are students able to understand the problem?***

What are the quantities in this problem?

> ***The number of pounds of vegetables (quantity #1) and the number of guests served (quantity #2).***

- Do the quantities vary?

  The quantities vary from one night to the next. As the number of guests increased from 12 guests on Friday to 28 guests on Saturday, the number of pounds of vegetables increased. As the number of guests decreased from 28 guests on Saturday to 20 guests on Sunday, the number of pounds of vegetables decreased.

- Do corresponding pounds of vegetables and number of guests seem to vary in some dependent way?

  The more guests that are served, the more vegetables are required.

- Given the context, how should the measures vary?

  As one measure increases, so does the other. We would expect a similar number of pounds of vegetables per guest, regardless of the day of the week.

- Does it look like twice the number of guests would require twice as many pounds of vegetables?

  Yes. Assuming that each guest is served the same amount of vegetables, it is reasonable to assume that twice the number of guests would require twice as many pounds of vegetables.

B. Content: Proportional Relationships (Meaning)

What is a proportional relationship?

> ***If a student says*** a/b=c/d***, more questions need to be asked of the student as to the meaning of*** a/b ***and*** c/d***. A proportional relationship is a relationship between measurable varying quantities such that pairs of measures scale in tandem. If the measure of one value changes by a factor of*** k***, then the other value also changes by a factor of*** k***.***
>
> ***Students who learn to identify related quantities that scale in tandem can recognize a proportional relationship. Note: In this item, the proportional relationship may be more evident through the constant ratio, but students should also be able to describe how the pairs of measures scale in tandem.***

C. Content: Proportional Relationships (Covarying quantities, representation)

In the image below, the length of the blue line increased as shown. If the length of the red line is proportional to the length of the blue line, what is the length of the longer red line? Can you explain it without using a formula?

> ***The length of the new blue line is $\frac{5}{3}$ the length of the original blue line. So, the length of the new red line must also be $\frac{5}{3}$ the length of the original red line, which is of 2, or $\frac{10}{3}$.***

- Are there any meaningful multiplicative comparisons in the above diagram?

  We can compare, multiplicatively, the length of the original blue line (3 units) and the increased length of the blue line (5 units). The increased length (5 units) is 5⁄3 (or 1 2⁄3) times the length of the original length (3 units). Equivalently, the original length (3 units) is 3⁄5 times the increased length (5 units). Alternatively, we can compare the length of the original blue line (3 units) with the length of the original red line (2 units) by claiming that the red lines' length is 2⁄3 times the length of the blue line. Reciprocally, the blue line's length (3 units) is 3⁄2 times the red line's length (2 units).

  - What is meaningful about the number 5⁄3 in the diagram above?

    It represents the ratio of the length of the longer blue line (5 units) to the length of the original blue line (3 units). It can be thought of as the scale factor between the lengths. It represents the multiplicative comparison of the increased length of the blue line (5 units) and the original length of the blue line (3 units).

  - What is meaningful about the number 2⁄3 in the diagram above?

    It represents the ratio of the length of the original red line (2 units) to the length of the original blue line (3 units). It can be thought of as the scale factor between the lengths. It represents the multiplicative comparison of the original length of the red line (2 units) and the original length of the blue line (3 units).

- If we imagine the blue line growing across all possible lengths, what happens with the red line? Are you sure?

  Like the blue line, the red line will grow in a way that keeps the ratio between the lengths constant. If the blue line doubles in length, so will the red line. If the blue line increases its length by a factor of 1.1 times, so will the red line. Likewise, the length of the red line will always be 2⁄3 the length of the blue line.

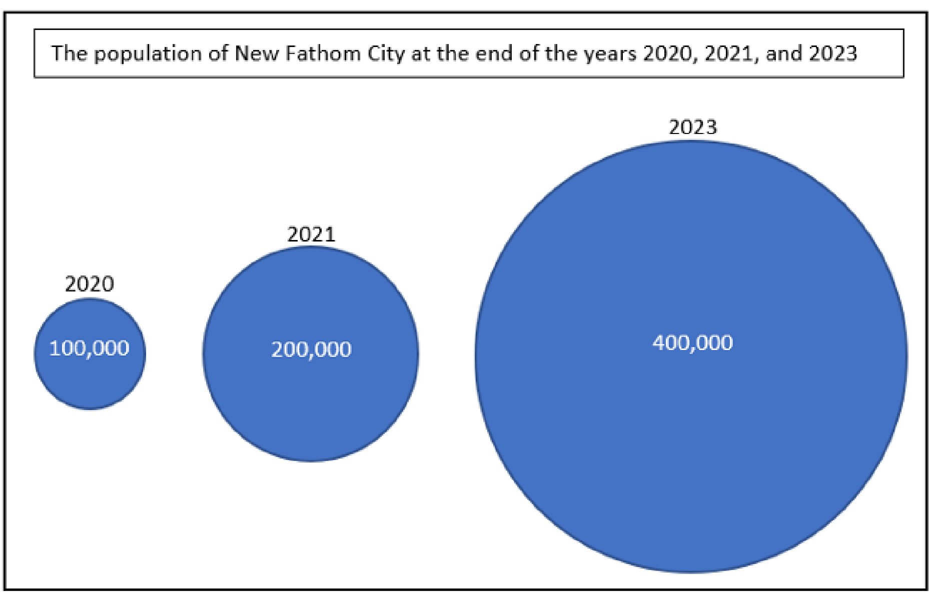
- Can you write an equation that shows the relationship between the lengths of the blue and red lines?

  If "b" is the length of the blue line and "r" is the length of the red line, then $r=2/3 \cdot b$. Or, $b=3/2 \cdot r$.

D. Content: Proportional Relationships (Meaning)

In the following graphic, tell me when you see two quantities that are in proportion to each other.

Students might note the quantities of years, population, circle areas, circle diameters, or circle circumferences. The circle diameters seem to be proportional to the populations. From 2020 to 2021 the population is doubled, and the diameter doubles as well. From 2020 to 2023 the population scales by 4 and so does the diameter. Since the circumference of a circle is proportional to the diameter ($C/d=\pi$), the population is also proportional to the circumference.



- Which pairs of quantities do not seem to be in a proportional relationship?

  It is likely that any pairs of quantities other than those mentioned above are not proportional. For example, doubling the population does not double the year, and doubling the population quadruples the area of the circle.

E. Problem Solving: Strategy

Go back to the original problem about the number of pounds of vegetables a restaurant needs for a certain number of guests. How could you find the number of pounds of vegetables that would likely be prepared for 4 guests?

From a scaling perspective, one might find a given number of guests that easily scales to 4. For example, 4 is $\frac{1}{5}$ of 20. So, if 25 pounds of vegetables corresponds to 20 guests, then $\frac{1}{5}$ of 25 pounds (or, 5 pounds) corresponds to $\frac{1}{5}$ of 20 guests.

If we think about maintaining a constant ratio, we see that the pounds of vegetables are always $\frac{15}{12}=\frac{35}{28}=\frac{25}{20}=1.25$ times the number of quests, meaning the pounds of vegetables should be $1.25\times4=5$.

- Can you think of another way to find the number of pounds of vegetables that would likely be prepared for 4 guests?

  Can the student invoke more than one of the above interpretations or perhaps something else?

- Can you figure out how many guests you could serve with 10 pounds of vegetables?

  Does the student see the opportunity to double the previous answer, or is there a need to restart?

- Can you think of another way to solve the problem?

  Does the student have multiple, flexible approaches?

F. Content: Proportional Relationships (Representation)

Here's one way to organize the data in the original problem:

Number of Guests	G	1	12	20	28
Pounds of Vegetables			15	25	35

If the relationship between guests and pounds of vegetables is proportional, how many pounds of vegetables does one guest receive?

The number of pounds of vegetables is always 1.25 times the number of guests, so 1.25. Or, I might scale the (12, 15) pair by $\frac{1}{12}$ to get $\frac{1}{12}\cdot15=\frac{15}{12}$.

- Using *G* as part of your answer, what belongs in the box below *G*?

  1.25G

- Is there an equation that would show that relationship?

  V=1.25G, or y=1.25x

- If there are 1.25, or $\frac{5}{4}$, pounds of vegetables for every guest, how many guests are there per pound of vegetables?

  Do students recognize the reciprocal relationship: $\frac{1}{1.25}$, or $\frac{4}{5}$?

G. Extension: Problem Solving

Suppose that in some situation apples are proportional to bananas and bananas are proportional to cherries. Does this mean apples are proportional to cherries? Explain.

Yes. For example, if I double the number of apples, I double the number of bananas. When I double the number of bananas, I double the number of cherries. Doubling the number of apples also means doubling the number of cherries, so apples are proportional to cherries.

7.1 Shareables*

This question has two parts. Use the information to answer Part A and Part B.

A restaurant owner keeps track of the food prepared for guests. On Friday, 15 pounds of vegetables were prepared for 12 guests. On Saturday, 35 pounds of vegetables were prepared for 28 guests. On Sunday, 25 pounds of vegetables were prepared for 20 guests.

Part A

Select one choice from the set to complete the sentence.

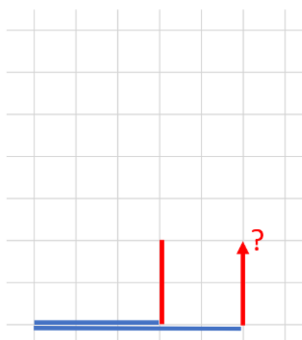
Based on this data, the number of pounds of vegetables **[is / is not]** proportional to the number of guests.

Part B

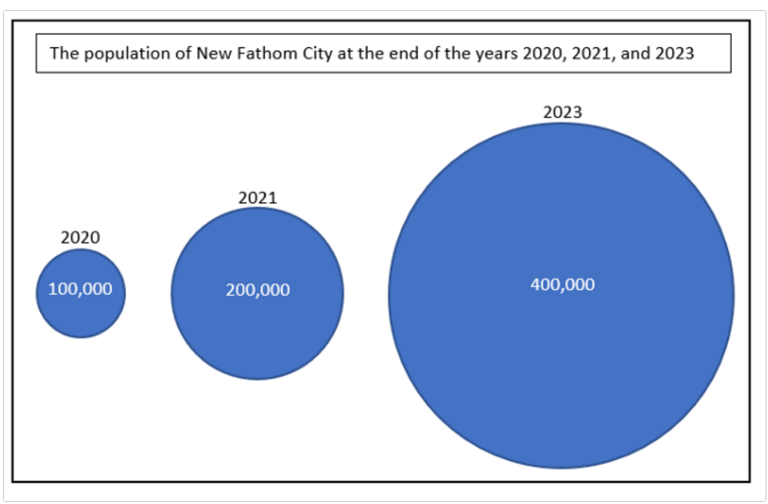
What is the constant of proportionality, if any, for this scenario?

- A. 1.25 guests per pound of vegetables
- B. 1.25 pounds of vegetables per guest
- C. there is no constant of proportionality

C.



D.



F.

Number of Guests	G	1	12	20	28
Pounds of Vegetables			15	25	35

7.2 Operations

This activity focuses on student thinking about multiplying and dividing rational numbers.

Operations: Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS 7.NS.A.2

While this item focuses on multiplying and dividing rational numbers, it also provides an opportunity to talk about representations, ordering, and meanings of operations.

THE CONVERSATION STARTER

Choose the expression in each row that has the greater value.

Row 1	$-\frac{3}{4}\times 2$	$-\frac{1}{2}\times 2$
Row 2	-1.5×20	-2×30
Row 3	$(-10)\div(-0.3)$	$(-10)\div(0.2)$

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation & Strategy

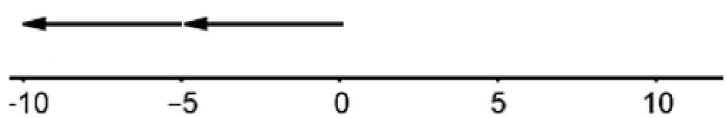
What do the directions mean by "greater value"?

- Which is greater: -¾ or -½? Why?
- Is knowing this enough to choose the expression that has the greater value in Row 1? Why?

B. Content: Operations (Multiplication, Meaning)

In mathematics, what does multiplication do for us?

- Think about 2×5 and 5×2. How are they different? How are they the same?
- Think about 2×(-5) and 5×(-2). How are they different? How are they the same?
  - How would you show 2×(-5) and (-2)×5 on the number line?



  - What real-life situation would result in needing to find 2×(-5)? How about (-2)×5?

Without doing the computation, what would you say (-1.5)×20 *means*?

C. Content: Operations (Division, Meaning)

In mathematics, what does division do for us?

- Without computing, what is one meaning of division you can use to make sense of 10÷(0.2) (or 10÷⅕)?
- Without computing, what is one meaning of division you can use to make sense of (-10)÷(0.2) (or (-10)÷⅕)?

Without computing, describe how you can estimate the value of (-10)÷(-0.3) using division as comparison (copies of, or times as large language).

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation & Strategy

What do the directions mean by "greater value"?

"Greater value" means the number that is the farthest right on the number line. If a student uses "largest value" or "biggest number," press them to use the number line to further demonstrate understanding.

- Which is greater: -¾ or -½? Why?

  -½ because it is the number farthest to the right on the number line.

- Is knowing this enough to choose the expression that has the greater value in Row 1? Why?

  Yes, both fractions are being multiplied by the same value, 2, which will not change the order of the fractions.

B. Content: Operations (Multiplication, Meaning)

In mathematics, what does multiplication do for us?

It counts the number of objects in equal-sized groups, gives area or items in an array, compares, or scales.

- Think about 2×5 and 5×2. How are they different? How are they the same?

  Example: If the first expression refers to two copies of five, then the other refers to five copies of two, yet both give a solution of ten. Or, we might think in a context: 2 hours at 5 miles per hour versus 5 hours at 2 miles per hour, yet both solutions result in 10 miles.

- Think about 2×(-5) and 5×(-2). How are they different? How are they the same?

  Both result in a solution of –10 but represent different situations.

  - How would you show 2×(-5) and (-2)×5 on the number line?

  - What real-life situation would result in needing to find 2×(-5)? How about (-2)×5?

    Possible solution for 2×(-5): If I spend five dollars each month for two months, how much does my account change? Possible solution for (-2)×5: The current temperature is zero degrees. The temperature drops two degrees each hour for 5 consecutive hours. What is the resulting temperature?

Without doing the computation, what would you say (-1.5)×20 *means*?

Thinking of multiplication as arrays, area, or repeated addition may not work so well here. It could, though, mean –1.5 copies of 20, where –1 copy of 20 is –20 and another half copy would be another –10.

C. Content: Operations (Division, Meaning)

In mathematics, what does division do for us?

> ***Division tells size of groups, or number of groups. It also provides multiplicative comparisons (telling times as large as).***

- Without computing, what is one meaning of division you can use to make sense of 10÷(0.2) (or 10÷⅕)?

  Possible solutions: How many ⅕'s are in 10, or 10 is how many times as large as ⅕, or 10 is ⅕ of what whole group.

- Without computing, what is one meaning of division you can use to make sense of (-10)÷(0.2) (or (-10)÷⅕)?

  Possible solutions: How many ⅕'s are in –10, or –10 is how many times as large as ⅕, or –10 is ⅕ of what whole group.

Without computing, describe how you can estimate the value of (-10)÷(-0.3) using division as comparison (copies of, or times as large language).

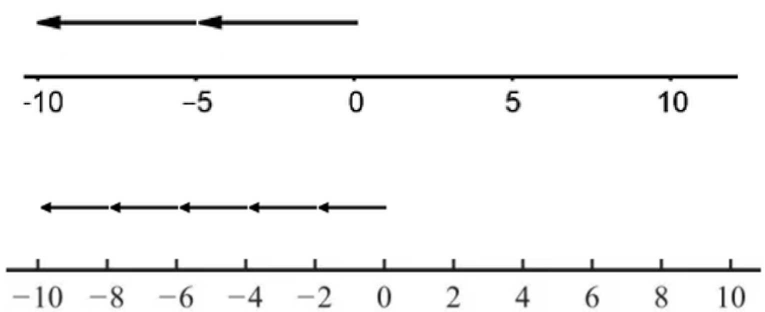
> ***The expression tells how many copies of (-0.3) there are in (-10). Given that 0.3 is close to ⅓, a ballpark estimate would be 10×⅓=30.***

7.2 Shareables*

Choose the expression in each row that has the greater value.

Row 1	$-\frac{3}{4}\times 2$	$-\frac{1}{2}\times 2$
Row 2	-1.5×20	-2×30
Row 3	$(-10)\div(-0.3)$	$(-10)\div(0.2)$

A.



7.3 Operations

This activity focuses on student thinking about operations with rational numbers in a real-world context.

Operations: Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS 7.NS.A.3

This item focuses on subtraction with rational numbers. It also provides an opportunity to talk about operation meanings and representations.

THE CONVERSATION STARTER

Use the information to answer the question.

The table shows the air temperature on the ground and the air temperature outside a plane when the plane is at cruising altitude.

Location	Air Temperature (°F)
Ground	53
Cruising Altitude	−64

How much colder is the air temperature at cruising altitude compared to the air temperature on the ground? Enter a number in the box to complete the sentence.

It is [ ] °F colder.

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

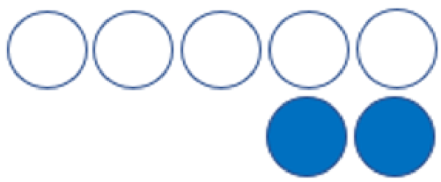
In your own words, what is the question asking?

- What does the negative number mean in the problem?

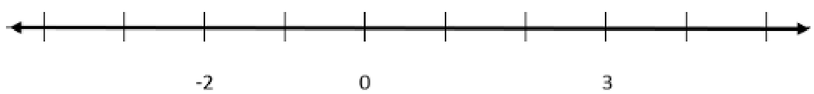
B. Content: Operations (Subtraction, Meaning)

In earlier grades, you learned about what subtraction means. What does it mean?

- Sometimes subtraction means to "take away." Tell me when you see how you could model 3 – (–2) as take-away in this diagram.
  - If needed: Suppose each blue chip represents –1 and each white chip represents 1.



- Sometimes subtraction means to "compare." Tell me when you see how you can model 3 -(-2) as a comparison on the number line.



C. Content: Operations (Subtraction, Comparison)

How are the expressions below alike? How are they different?

$$50 - (-60) \qquad (-60) - 50$$

How are the expressions below alike? How are they different?

$$A + B \qquad A - (-B)$$

D. Content: Operations (Subtraction, Meaning)

What way of thinking about subtraction helps you make sense of 53 – (–64)?

- What way of thinking helps you make sense of (–64) – 53?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

In your own words, what is the question asking?

For example: What is the difference between the two temperatures? How much warmer is the temperature on the ground compared to the temperature at cruising altitude?

- What does the negative number mean in the problem?

It means that the temperature is far below 0°F, so it is very cold.

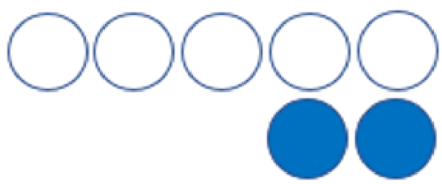
B. Content: Operations (Subtraction, Meaning)

In earlier grades you learned about what subtraction means. What does it mean?

Subtraction can be demonstrated in a "take away" situation or in a "comparison" situation.

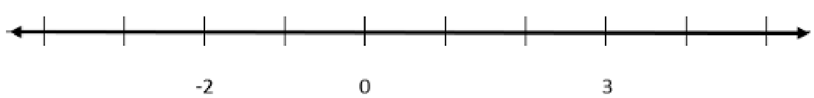
- Sometimes subtraction means to "take away." Tell me when you see how you could model 3 – (–2) as take-away in this diagram.
  - If needed: Suppose each blue chip represents –1 and each white chip represents 1.

One might think of the white chips as each having a value of +1 while the blue chips each have a value of –1. With that, the diagram as given represents +3. If you take away two blue chips (2×(-1)=-2), you are left with an overall value of 5.



- Sometimes subtraction means to "compare." Tell me when you see how you can model 3 -(-2) as a comparison on the number line.

To compare 3 to –2 is to determine how many units 3 is from –2 and in which direction. Since 3 is 5 units to the right of –2, we can say that 3 – (–2) = 5.



C. Content: Operations (Subtraction, Comparison)

How are the expressions below alike? How are they different?

$$50 - (-60) \qquad (-60) - 50$$

They both include the numbers 50 and –60. They both involve subtraction. They both can be thought about as a comparison of two quantities. However, in 50 – (–60), we can think, "How far is 50 from –60 on a number line and in what direction?" We know 50 is 110 units to the right of –60, so the result is positive 110 from a comparison perspective. In (–60) – 50, we can compare the quantities by saying that –60 is 110 units to the left of 50 on a number line and so the result is –110.

How are the expressions below alike? How are they different?

$$A + B \qquad A - (-B)$$

They are the same in that they produce the same numeric answer. They are different in that they have different ways of thinking associated with them: For addition, I think about putting things together. In the case of subtraction, I am either comparing (such as finding a distance on a number line) or taking things away. Note: If a student solves the problem by indicating that "two negatives make a positive" or by using a rule like "you chop-chop," ask the student to explain how the result makes sense in terms of the meanings of addition and subtraction.

D. Content: Operations (Subtraction, Meaning)

What way of thinking about subtraction helps you make sense of 53 – (–64)?

Using a number line, perhaps: 53 – (–64) means, "How far is 53 from –64 and in what direction?" 53 is 117 units to the right of –64 on the number line, so 53 – (–64) = 117.

- What way of thinking helps you make sense of (–64) – 53?

It might mean, "How far is –64 from 53 and in what direction?" We know –64 is 117 units to the left of 53 on the number line, so –64 – (53) = –117.

7.3 Shareables*

Use the information to answer the question.

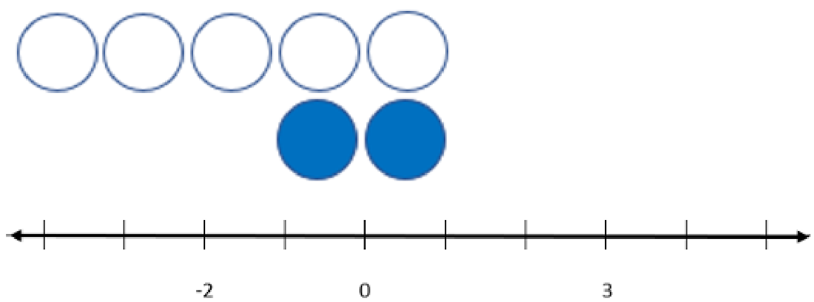
The table shows the air temperature on the ground and the air temperature outside a plane when the plane is at cruising altitude.

Location	Air Temperature (°F)
Ground	53
Cruising Altitude	−64

How much colder is the air temperature at cruising altitude compared to the air temperature on the ground? Enter a number in the box to complete the sentence.

It is ☐ °F colder.

B.



C.

$$50 - (-60) \qquad (-60) - 50$$

$$A + B \qquad A - (-B)$$

7.4 Operations

This activity focuses on student thinking about solving real-life problems with positive and negative rational numbers.

Operations: Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS: 7.EE.B.3

This item focuses on solving real-life problems with rational numbers. However, it also provides an opportunity to talk about percentages, expressions, and scaling in tandem.

THE CONVERSATION STARTER

Use the information to answer the question.

A toy store is having a sale on puzzles. For each puzzle Naomi buys she gets 75% off a second puzzle. The original price of each puzzle is $\$16.50$.

How much would 4 puzzles cost? Enter the answer in the box.

$ [ ]

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation & Strategy

Can you describe what's going on in this problem in your own words?

In the original problem, if someone buys 1 puzzle, how much would they pay?

- If someone buys 2 puzzles, how might you **estimate** how much will they pay?

B. Content: Percent (Meaning, Representation)

Let's talk about percentages. What is a percent?

- How can you represent 75% as a decimal? Why does that make sense?
- What are the different ways someone could find 45% of 50?

Can you explain how scaling in tandem can help you find 40% of 50?

  - If needed: What is 10% of 50?

- How can someone find 12% of a number in their head?
  - Explain it to me by finding 12% of 50.

In general, what does 130% mean?

- How can you write 130% as a decimal? Why does that make sense?
- How could someone figure out 130% of 50 in their head?
- Is 130% even possible in real life? Some people say 100% is everything so you cannot have more than that.

C. Content: Expressions (Representation)

Suppose I tell you that after paying 6% tax I ended up paying $135.26 for a new jacket. In this context, what might each of the following computations tell me:

$$0.06(135.26)$$

D. Content: Expressions (Meaning)

In terms of percentages, what would these expressions tell us?

$$1.30x - 0.9x$$

$$(1.3x)(0.9)$$

- Tell me when you see how one of those expressions represents an increase of 30% in some quantity, which is then followed by a decrease of 10%.

E. Content: Percent (Application, Meaning)

In the original problem, what overall percentage discount do you really get when you buy two puzzles?

- What overall percentage discount do you really get when you buy four puzzles? Why?
- Will the overall percentage discount on 5 puzzles be greater or less than the overall percentage discount on 4 puzzles?
- When is the following statement true? When is it false?

  In a sale where I buy one and get another of equal or less value at 50% off, I really save 25% overall when I buy 2.

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation & Strategy

Can you describe what's going on in this problem in your own words?

> ***Naomi is buying 4 puzzles. Two of the puzzles will cost the original price of $16.50. Two of the puzzles will be 75% off or 25% of $16.50. What will be the total cost of purchasing 4 puzzles?***

In the original problem, if someone buys 1 puzzle, how much would they pay?

> ***$16.50***

- If someone buys 2 puzzles, how might you **estimate** how much will they pay?

  The first puzzle costs $16.50. The discount of 75% on the second puzzle will result in a cost of about $4 (since a quarter of $16 is $4). We can estimate the cost (before tax) to be about $20.

B. Content: Percent (Meaning, Representation)

Let's talk about percentages. What is a percent?

> ***A percent is a multiplicative comparison of one quantity to another that is interpreted in terms of "per 100." We view the compared value to the whole as if the whole is scaled to a value of 100. 1% is $\frac{1}{100}$ of a quantity. 50% is 50 copies of $\frac{1}{100}$ of a quantity.***

- How can you represent 75% as a decimal? Why does that make sense?

  When we say or write 75%, we mean 75 copies of $\frac{1}{100}$ or $\frac{75}{100}$. Therefore, 75% can be expressed by the decimal number of 0.75 which is also 75 hundredths.

- What are the different ways someone could find 45% of 50?

  One strategy is to first find 10% of 50, which is 5. Four copies of this (4 x 5 = 20) will get us to 40% of 50. Since 5% is half of 10%, we can add an additional 2.5 (half of 5 or 5% of 50) to obtain 22.5. So, 45% of 50 is 22.5. Another strategy could be to notice that 50 is half of 100, so 45% of 50 is half of 45% of 100, giving 22.5. Another strategy could be to notice that I need $\frac{45}{100}$, or 0.45, copies of 50, which can be found by multiplying 0.45 x 50.

Can you explain how scaling in tandem can help you find 40% of 50?

> ***One can start by recognizing that 10% of 50 is 5. We need 4 times as much as 10% so we need 4 times as much as 5.***

  - If needed: What is 10% of 50?

    10% of 50 is 5.

- How can someone find 12% of a number in their head?

  Start by finding 10% of the number. Then, $\frac{1}{10}$ of that amount is 1%. Add the result of finding 10% of a number to 2 copies of 1% of a number.

  - Explain it to me by finding 12% of 50.

    Start by finding 10% of 50, which is 5. Then, $\frac{1}{10}$ of 5 is $\frac{1}{2}$. That means, $\frac{1}{2}$ is 1% of 50. Add the 5 (10% of 50) to 2 copies of $\frac{1}{2}$ (1% of 50). That is, $5 + 2 \cdot \frac{1}{2} = 6$. Therefore, 12% of 50 is 6.

In general, what does 130% mean?

> ***As a fraction, it represents $\frac{130}{100}$. We can think of $\frac{130}{100}$ as 130 copies of $\frac{1}{100}$ where $\frac{1}{100}$ is obtained by cutting a whole unit up into 100 equal parts. Therefore, 130% or $\frac{130}{100}$ consists of 1 whole unit, or 100 copies of $\frac{1}{100}$, plus an additional $\frac{30}{100}$ of a whole unit.***

- How can you write 130% as a decimal? Why does that make sense?

  We can think of 130% as 130 copies of $\frac{1}{100}$, or $\frac{130}{100}$, where $\frac{1}{100}$ is obtained by cutting a whole unit up into 100 equal parts. Therefore, 130% or $\frac{130}{100}$ consists of 1 whole unit, or 100 copies of $\frac{1}{100}$, plus an additional $\frac{30}{100}$ of a whole unit. This is expressed using the decimal number 1.30.

- How could someone figure out 130% of 50 in their head?

  100% of 50 is 50. Then, 10% of 50 is 5. Three copies of 10% or three copies of 5 is 15. Therefore, 130% of 50 is 50 + 15 = 65.

- Is 130% even possible in real life? Some people say 100% is everything so you cannot have more than that.

  Yes. Suppose some land is owned as an investment property. The value of this property can increase such that it is currently worth 130% of what it was worth in some previous year. Suppose the initial investment was $50,000. This property could have grown in value such that it is worth 130% of $50,000 or $65,000.

C. Content: Expressions (Representation)

Suppose I tell you that after paying 6% tax I ended up paying $135.26 for a new jacket. In this context, what might each of the following computations tell me:

$$0.06(135.26)$$

This is an expression representing 6% of the total cost (including tax) of purchasing the jacket. It would represent the calculation of an additional 6% tax. It would not be relevant to the situation.

$$1.06(135.26)$$

This is an expression representing 106% of the total cost (including tax) of purchasing the jacket. It would represent what would happen if we added another 6% to the total. It would not be relevant to the situation.

$$\frac{135.26}{1.06}$$

This is an expression representing the price, before tax, of the new jacket. It tells us the number (before-tax price of the new jacket) that when multiplied by 1.06 is 135.26.

$$135.26 - 0.06$$

This is an expression representing a decrease of $0.06 or 6 cents from the total cost (including tax) of purchasing the jacket. It would not be relevant to the situation.

D. Content: Expressions (Meaning)

In terms of percentages, what would these expressions tell us?

$$1.30x - 0.9x$$

$$(1.3x)(0.9)$$

The first is an expression that could represent 40% of a quantity, x. Note that one could think of this equivalently as (1.30 – 0.9)x = (0.4)x.

The second is an expression that could represent an increase of 30% (1.3x) followed by finding 90% (0.9) of that increased quantity. Finding 90% of a quantity is equivalent to decreasing that quantity by 10%.

- Tell me when you see how one of those expressions represents an increase of 30% in some quantity, which is then followed by a decrease of 10%.

  The second expression represents this. For example, suppose $100 as the quantity, then (1.3)(100) = 130, which is an increase of 30%, which is then followed by a decrease of 10%, (130)(0.9) = 117. 130 – 117 = 13, which is the 10% decrease.

E. Content: Percent (Application, Meaning)

In the original problem, what overall percentage discount do you really get when you buy two puzzles?

37.5%. Because the puzzles cost the same amount, you can think of splitting the discount equally between the two.

- What overall percentage discount do you really get when you buy four puzzles? Why?

  It's the same as with two puzzles. If I save 37.5% on the first pair and 37.5% on the second pair, I save 37.5% overall.

- Will the overall percentage discount on 5 puzzles be greater or less than the overall percentage discount on 4 puzzles?

  When I include the odd-numbered puzzle, the overall discounted amount does not increase, yet the overall cost goes up. As such, the overall percentage discount goes down.

- When is the following statement true? When is it false?

  In a sale where I buy one and get another of equal or less value at 50% off, I really save 25% overall when I buy 2.

  It is true when the two items have the same price. For example, if both items cost $20, then I will pay $30, instead of $40, for two items. Since $10 is 25% of $40, I save 25% overall. However, if one item costs less than the other, such as if one item costs $40 and the other $10, then I will pay $40 + $5 = $45, instead of $50. That's only an overall savings of 10%.

7.4 Shareables*

Use the information to answer the question.

A toy store is having a sale on puzzles. For each puzzle Naomi buys she gets 75% off a second puzzle. The original price of each puzzle is $\$16.50$.

How much would 4 puzzles cost? Enter the answer in the box.

$ [ ]

C.

$$0.06(135.26)$$

$$\frac{135.26}{1.06}$$

$$1.06(135.26)$$

$$135.26 - 0.06$$

D.

$$1.30x - 0.9x$$

$$(1.3x)(0.9)$$

7.5 Formulas

This activity focuses on student thinking about probability and probability models.

Formulas: Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., A = πr2 means three-and-a-bit copies of the square with area r2).

ITEM ALIGNMENT

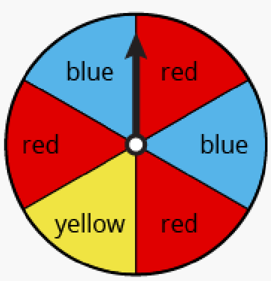
CCSS: 7.SP.C.7

This item focuses on probability and probability models. However, it also provides an opportunity to talk about operations with fractions, percentages, and fairness.

THE CONVERSATION STARTER

Use the information to complete the task.

A spinner is divided into 6 equal sections and colored as shown. The spinner was spun 60 times, and the results are recorded in the table.



Red	Blue	Yellow
25	12	23

Compare the number of times the spinner landed on each color to the expected number of times the spinner should have landed on each color. Select "more" or "fewer" to complete each statement.

The spinner landed on red **[more / fewer]** times than expected for a fair spinner.
The spinner landed on blue **[more / fewer]** times than expected for a fair spinner.
The spinner landed on yellow **[more / fewer]** times than expected for a fair spinner.

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

In your own words, what is this question asking?

- Is this a probability question? Why?
- What is probability?
- What kinds of numbers show up in probability? Can you give some examples?

B. Content: Probability (Meaning, Equally Likely)

What does "expected for a fair spinner" mean in this question?

- Without looking at the results, if you spun the spinner 100 times, what would you expect to see for the number of times it stops on red?
  - Would you really expect that, or would you expect something *close* to that?
  - How close to that do you think you would be, really?
  - What if you spun it another 100 times? Would you expect the same answer you just gave me? Why or why not?
- Is the spinner in this problem fair?
  - If I flipped a fair coin 10 times, how many times would you expect it to land on heads?
  - Could a fair coin *never* land on heads in 10 flips?
  - Suppose I flip a fair coin 9 times and I get 9 heads. What is the probability I will get heads on the next flip?

Red	Blue	Yellow
24	23	13

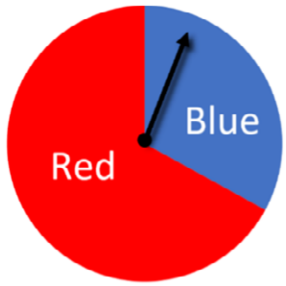
- Imagine you spun the spinner another 60 times and got the results shown in the table:
  - Could that really happen?
  - What would you want to do next?

C. Content: Probability (Meaning)

How are probabilities and percentages related? How are they different?

- Consider the number 1.3.
  - What would it mean as a probability?
  - What would it mean as a percent?

Extension: Given the following spinner, how might you predict the probability of getting a red and then a blue on consecutive spins?



CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

In your own words, what is this question asking?

One possible answer: I need to determine how these results compare to what we expect from a fair spinner.

- Is this a probability question? Why?

  Yes, because I need to consider the likelihood of each chance event (color) as a number between 0 and 1 in order to answer it.

- What is probability?

  The Common Core State Standards define probability as follows: "The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event."

  That is, probability is the likelihood of something happening. It is a number between 0 and 1, though it might be represented as a fraction, decimal, or percent. For example, if a coin is flipped there are two equally likely outcomes: heads or tails. Heads is one of the outcomes, so the likelihood of a flipped coin landing on heads is ½. Thus, the probability (likelihood) of a flipped coin coming up as heads can be represented as ½, 0.5, or 50%.

- What kinds of numbers show up in probability? Can you give some examples?

  A reported probability must be a number between 0 (impossible) and 1 (certain). For example, with a probability of 1, a fair coin that is flipped into the air will land on either heads or tails. If a card is randomly selected from a standard deck, the probability of it being the 5 of hearts is 1⁄52.

B. Content: Probability (Meaning, Equally Likely)

What does "expected for a fair spinner" mean in this question?

It would mean that landing on each area of ⅙ of the spinner is equally likely. It would mean that the fractions of areas for the colors match up with the fractions of occurrences when the spinner lands on them. If a color is ⅙ of the area, we expect the spinner to land on that color ⅙ of the time.

- Without looking at the results, if you spun the spinner 100 times what would you expect to see for the number of times it stops on red?

  Since 3⁄6 or ½ of the possible spaces are red, we expect ½ of the 100 spins to stop on red. That is, we expect red to occur 50 times.

  - Would you really expect that, or would you expect something close to that?

    We would expect something close to 50 outcomes of stopping on red.

  - How close to that do you think you would be, really?

    Answers may vary, but 100 spins might be large enough for the results to be close to the expected outcomes. However, in an individual experiment involving 100 spins, it is possible for the results to be, shall we say, surprising!

  - What if you spun it another 100 times? Would you expect the same answer you just gave me? Why or why not?

    No, each experiment involving 100 spins could have different results.

- Is the spinner in this problem fair?

  Well, there are two notions here—fair with respect to the areas of the colors or fair with respect to the outcomes when spun 60 times. On the first, there is more red area, so in this manner the spinner would not be considered fair.

With respect to outcomes, for example, we might expect ⅙, or 10, of the spins to be yellow, but we ended up with 23. Therefore, we have some serious doubt that the spinner is fair, but this could also be said to have happened by chance.

- If I flipped a fair coin 10 times, how many times would you expect it to land on heads?

  With a probability of ½, it is expected that the coin will land on heads 5 times. However, we might not be surprised to find the results to be such that the number of heads is a little more or a little less than 5 times.

- Could a fair coin never land on heads in 10 flips?

  Yes. A fair coin could have 10 tails in a row and still be fair. However, it is not very likely!

- Suppose I flip a fair coin 9 times and I get 9 heads. What is the probability I will get heads on the next flip?

Red	Blue	Yellow
24	23	13

  The probability of obtaining heads for any individual flip is ½.

- Imagine you spun the spinner another 60 times and got the results shown in the table:
  - Could that really happen?

    Yes, it could really happen.

  - What would you want to do next?

    Try another experiment with 60 spins to see if the results are more aligned with the expected probabilities. If we consistently obtained results that were far off from the expected probabilities, we might conclude that the spinner is not fair.

C. Content: Probability (Meaning)

How are probabilities and percentages related? How are they different?

Both are based on a whole of 1. However, percentages can exceed a whole (greater than 100%) and probabilities cannot (they must be greater than or equal to 0 and less than or equal to 1).

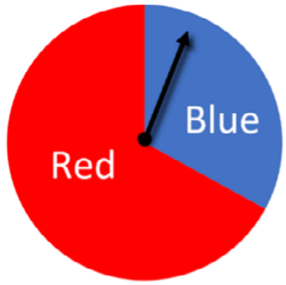
- Consider the number 1.3.
  - What would it mean as a probability?

    It would be impossible, as an outcome cannot occur more than 100% of the total number of possible outcomes.

  - What would it mean as a percent?

    It would mean 130%, which is entirely possible. Suppose you bought land for $50,000. If the property is now valued at $65,000, it is worth 130% of the purchase price.

Extension: Given the following spinner, how might you predict the probability of getting a red and then a blue on consecutive spins?

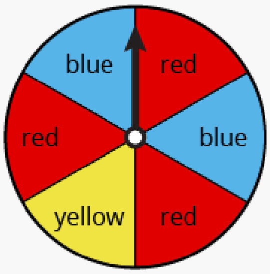


On the first spin, we expect red to occur roughly $\frac{2}{3}$ of the time. Of these $\frac{2}{3}$ of the spins, $\frac{1}{3}$ of those spins will land on blue on the next spin. So, the probability of red followed by blue is $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$. A tree diagram can clearly show the possibilities for a red followed by a blue.

2.5 Shareables*

Use the information to complete the task.

A spinner is divided into 6 equal sections and colored as shown. The spinner was spun 60 times, and the results are recorded in the table.



Red	Blue	Yellow
25	12	23

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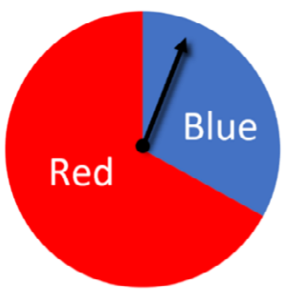
The spinner landed on red **[more / fewer]** times than expected for a fair spinner.
The spinner landed on blue **[more / fewer]** times than expected for a fair spinner.
The spinner landed on yellow **[more / fewer]** times than expected for a fair spinner.

B.

Red	Blue	Yellow
24	23	13

C.



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