Formative Conversation Starters: Math

GRADE 2

nwea Professional Learning

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Formative Conversation Starters

Student understanding is more about growing than it is about getting. As educators, we might speak of some students "getting it" and other students not. This conclusion, though, is not fair to students. A student who may not seem to "get it" does understand *something*, even if that *something* may not yet have grown into a robust web of thinking. To reach students' full knowledge, we can use guided conversations: Formative Conversation Starters.

Purpose: The purpose of Formative Conversation Starters is to help teachers reveal student understanding about key ideas in mathematics and to identify their students' ways of thinking.

Audience: The intended audience is teachers, who will use these questioning strategies with students.

Application: Teachers may wish to use these conversation starters in one-on-one conferences with students or in small groups.

Formative Conversation Starters approach student knowledge by presenting a single standards-based assessment item and leveraging the item to elicit conversation through clustered questioning. The goal of this activity is not to tell students what to think, but to help teachers better uncover how students are currently thinking about mathematical concepts. The conversations provide opportunities for students to communicate how they are thinking about mathematics.

Mathematical Ideas (BINSS – Big Ideas to Nurture Standards Sense-making)

As you read through these items, you will notice that we draw attention to a few specific mathematical ideas. These ideas correspond to important ways of thinking that all students should develop and continue to refine. They include:

- **Operations:** Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

- **Place Value:** Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (*or a bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

- **Comparisons:** Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is <u>3 times as large</u> as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is <u>4 meters more</u> than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

- **Measurement:** Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

- **Fractions:** A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

- **Formulas:** Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r^2$ means three-and-a-bit copies of the square with area r^2).

- **Variables:** Students should have ways of thinking that enable them to distinguish between unknowns that vary and unknowns that represent some fixed value. For example, in the equation $6=3x+2$, x represents some unknown fixed value that makes the equation true. In $y=3x+2$, y and x vary with each other. In $y=mx+b$, y and x vary with each other, while m and b are typically nonvarying constants (parameters) within a problem.

- **Covarying Quantities:** Single quantities can vary, but students also need to consider situations where two quantities vary together. For example, in the equation $y=3x+2$, as the quantity x varies, the quantity y varies. It can be helpful to think about the relationship between covarying quantities in terms of how changes in x result in changes in y (e.g., as x increases by 1, y changes by . . .).

- **Proportional Relationships:** Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

- **The Equal Sign:** The equal sign works in multiple ways in mathematics, even though the symbol does not change. Students should be able to think about the symbol as being relational (e.g., 2+4=5+1) and as being operational (e.g., the output of a computation), and should be able to determine which role the symbol is playing based on the situation. Keep an eye out for when students may put together equal signs. If this happens, prompting the student with questions about what the equals sign means may help. For example, suppose a student writes: 9 x 8 = 72 = 126 – 72 = 54 (*in this case, the student might be using the equal sign to say "next, I will…"*).

In the Formative Conversation Starters, you will notice that we call out mathematical ideas and the ways of thinking associated with them when it makes sense to do so. We also group discussion prompts to focus the conversations. Some groupings target core understandings that underpin the content. Other groupings elicit flexibility of thinking or extend beyond the assessment item being discussed. All of these methods are intended to provide opportunities for the teacher to listen to students and to reflect on how students might be thinking about mathematics in the standards-based assessment item.

The progressive question-and-answer strategy can be used to elicit evidence of students' ways of thinking about a topic or concept, with the purpose of guiding instruction.

How to Conduct a Formative Conversation

1. The questions for each item were developed to help teachers elicit information about students' ways of thinking about the content in the item and about mathematical ideas. These questions are suggestions, however, and not intended to be used as a script. The conversations teachers have will vary by student. While the questions for an item are laid out in a progression, teachers should vary the order to adapt to students' responses. Teachers should also keep in mind that students' responses may point to ways of thinking that are not addressed by the provided questions. In these cases, teachers should pursue those student understandings with their own line of questioning. There are several actions teachers can take to prepare for formative conversations:

    a. Become very familiar with the task and the questions ahead of time. This ensures that teachers can select the most appropriate next question based on how the students are responding.

    b. Provide students with tools to help them answer the questions. Depending on the task, these tools might include a manipulative, drawing paper, graph paper, or individual whiteboards and markers.

  c. Have a list of questions to help further probe what students are thinking. Some examples are:
     i. Can you tell me more about that?
     ii. You look like you're really thinking about this. What are you thinking?
     iii. Can you draw me a picture/write an equation?
     iv. How did you get that answer?
     v. Is there another way that you could find that answer?
  d. Make a plan to track what students say during the conversation: record the conversation, take notes, or have an observer take notes.

2. Teachers should ask questions without judgment. Student responses should not be labeled as right or wrong, and follow-up questions should be asked regardless of whether students give a correct response. Teachers should avoid commenting on students' responses other than to ask follow-up questions or to clarify what a student has said. Other students, however, should be encouraged to agree or disagree in a small group setting.

3. One of the most important parts of the formative conversation is what comes after the conversation: how will a teacher use the information about student thinking when planning instruction. Consider these suggestions for how to act on a formative conversation:
   a. Identify the different mathematical ideas addressed in the formative conversation. Where did students make connections between the ideas? Where do the connections need to be strengthened?
   b. Identify what students already understand in order to build instruction on that understanding.
   c. Identify areas where students can deepen what they already understand.
   d. Identify ways that students are comfortable with expressing mathematical ideas, and plan how to expand their capabilities. How were students most comfortable expressing or explaining what they understand? Were they more inclined to create a graphical representation of their thinking? Did they prefer to explain verbally? Did they use equations, or did they prefer to use graphs?

4. Some Conversation Starters include extension questions. These questions are provided as a way to elicit thinking beyond the grade-level of the BINSS.

2.1 Comparisons and Operations

Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is 3 times as large as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is 4 meters more than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS: 2.OA.A.1

TX: 2.7.C

VA: 2.6.C

This item focuses on understanding the operations of addition and subtraction and extends the work of grade 1, where one-step problems are used. However, it also provides an opportunity to talk about the concept of the "unknown" in all positions as students develop the meaning of addition and subtraction.

THE CONVERSATION STARTER

Use the information to answer the question.

Hannah has 12 fewer stickers than Emma. Hannah has 26 stickers. How many stickers does Emma have?

Which equation can be used to find out how many stickers Emma has?

- A. $12 + \square = 26$
- B. $12 + 26 = \square$
- C. $26 - \square = 12$
- D. $26 - 12 = \square$

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

- Who has more stickers? How do you know?

Do you have to take time to think about this question? Why do you think that is?

B. Problem Solving: Strategy

Let's ignore the answer options. How do you want to answer the question?

- Could you answer the question in your head, without writing anything down? If so, how would you start?
- Could you answer the question with a drawing? If so, how would you start?
- Could you answer the question with objects? If so, how would you start?
- Could you answer the question with an equation? If so, how would you start?

C. Content: Operations (Subtraction, Inverse Operations)

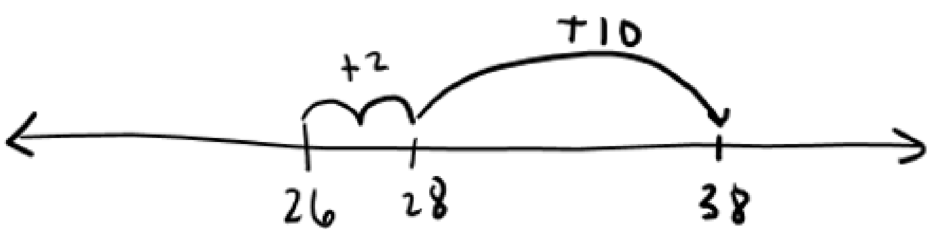
Could you use addition to describe the situation?

- Could you use subtraction to describe the situation?

D. Content: Operations (Subtraction, Meaning, Unknowns)

If we subtracted 26 − 12, what would that tell us in this situation?

Tell me when you see how this diagram could explain the situation in the question.



- Tell me when you see each of the following equations with unknowns in the diagram.

$$26 + 12 = \square$$

$$\square - 12 = 26$$

$$26 + \square + \square = \square$$

E. Content: Equality (Unknowns)

Let's look at some other equations with unknowns.

$$2 + 9 = \square$$

$$5 + \square = 8$$

$$2 + 6 = \square + 3$$

- What does the box, or unknown, mean in each equation?
- What does the equal sign mean in each equation? Are the three equal signs different in any way?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

> ***It is asking how to set up a math sentence to find how many stickers Emma has.***

- Who has more stickers? How do you know?

  Emma has more because the first sentence says that Hannah has 12 fewer stickers.

Do you have to take time to think about this question? Why do you think that is?

> ***Students may respond that they have to figure out what numbers they know, what they are trying to find, and what operation they need to use. The word "fewer" may also be tricky and initially lead students to think about subtracting 12 from 26.***

B. Problem Solving: Strategy

Let's ignore the answer options. How do you want to answer the question?

> ***Listen to how students want to approach problems like this. Some may want to think quietly, some may want to draw a picture. Based on how students answer, ask questions to gauge student flexibility with other approaches.***

- Could you answer the question in your head, without writing anything down? If so, how would you start?

  If Hannah has 12 fewer stickers than Emma, and Hannah has 26 stickers, add 26 and 12 to find how many stickers Emma has.

- Could you answer the question with a drawing? If so, how would you start?

  Draw Hannah's 26 stickers. Then think about how to show that Hannah has 12 fewer stickers than Emma, such as draw 12 more stickers. A number line is another way to draw to help answer the question.

- Could you answer the question with objects? If so, how would you start?

  Start with 26 objects to represent Hannah's stickers. Then think about how to show that Hannah has 12 fewer stickers than Emma, such as add 12 more objects.

- Could you answer the question with an equation? If so, how would you start?

  Start with 12 because Hannah has 12 <u>fewer</u> stickers than Emma. Then think about what the 26 means. Since 26 means the number of stickers Hannah has, and Emma has more stickers than Hannah, use addition to find how many stickers Emma has. For example, 12 + 26 = ☐.

C. Content: Operations (Subtraction, Inverse Operations)

Could you use addition to describe the situation?

> ***Yes, start with 26 and add 12 to find the unknown. 12 + 26 = ☐***

- Could you use subtraction to describe the situation?

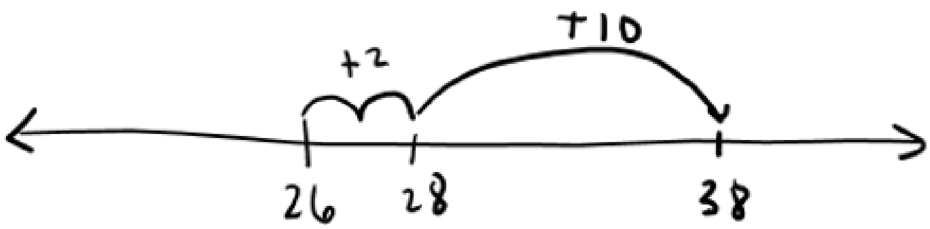
  Yes, start with the unknown, Emma's stickers, and then subtract 12, the number of stickers fewer that Hannah has. Set that equal to equal 26, the number of stickers Hannah has. ☐ – 12 = 26

D. Content: Operations (Subtraction, Meaning, Unknowns)

If we subtracted 26 – 12, what would that tell us in this situation?

It would tell us how many stickers someone with 12 stickers less than Hannah has. That person wouldn't be Emma because Emma has 12 more stickers than Hannah.

Tell me when you see how this diagram could explain the situation in the question.



In Grade 2, the open number line can be a tool/strategy for thinking about such problems. In this case, you might add 12 to 26 by first adding 2, then adding 10.

- Tell me when you see each of the following equations with unknowns in the diagram.

$$26 + 12 = \square$$

$$\square - 12 = 26$$

$$26 + \square + \square = \square$$

In the first equation, we have 12 more than 26, which is 38. In the second equation, we look for a number, that when subtracted from 12 results in 26, which is 38. The third equation allows for the multiple jumps on the number line.

E. Content: Equality (Unknowns)

Let's look at some other equations with unknowns.

$$2 + 9 = \square$$

$$5 + \square = 8$$

$$2 + 6 = \square + 3$$

- What does the box, or unknown, mean in each equation?

  It represents a value we don't know. In the first equation, the box is equal in value to the sum of 2 and 9. In the second equation, the box is the value that when increased by 5, equals 8. In the last equation, the box represents the value that when increased by 3, is equal to 2 + 6.)

- What does the equal sign mean in each equation? Are the three equal signs different in any way?

  In each of the equations, the equal sign is a statement that one side is equal in value to the other side. Be sure that students can flexibly interpret the equal sign as relational. A purely operational perspective, such as "the result when you add… ," will lead to difficulty when looking at the third equation, for example.

2.1 Shareables*

Use the information to answer the question.

Hannah has 12 fewer stickers than Emma. Hannah has 26 stickers. How many stickers does Emma have?

Which equation can be used to find out how many stickers Emma has?

- A. $12 + \square = 26$
- B. $12 + 26 = \square$
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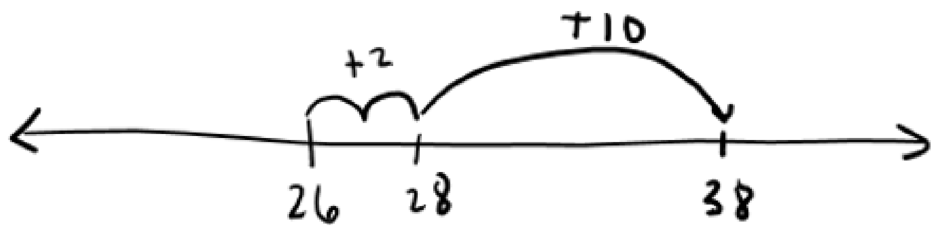
D.



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E.

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2.2 Comparisons and Operations

Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is 3 times as large as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is 4 meters more than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

ITEM ALIGNMENT

CCSS: 2.OA.A.1

TX: 2.4.C

VA: 2.6.C

This item focuses on understanding the relationship between quantities in a real-world situation. However, it also provides an opportunity to talk about applying operations to a real-world situation.

THE CONVERSATION STARTER

Use the information to answer the question.

A baker has 36 cherry pies and some apple pies. In all, there are 53 pies. The baker then bakes more apple pies. There are now 42 apple pies.

How many more apple pies did the baker bake? Enter the answer in the box.

[ ] pies

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

- Do you have to take time to think about this question? Why do you think that is?

B. Problem Solving: Strategy

What is the first thing you want to do? Why?

- Could you answer the question in your head, without writing anything down? If so, how would you start?
- Could you answer the question with a drawing? If so, how would you start?
- Could you answer the question with objects? If so, how would you start?
- Could you answer the question with equations? If so, how would you start?

C. Content: Operations (Addition, Subtraction, Meaning)

Let's talk about adding and subtracting.

When is it OK to add two numbers together? When is it not OK?

- Can I add a given quantity of apples to a given quantity of oranges?
- Can I add a given quantity of fish in the ocean to a given quantity of stars in space?

When is it OK to subtract two numbers? When is it not OK?

- Can I subtract a given quantity of apples from a given quantity of oranges?
- Would there ever be a reason to subtract a given number of fish in the ocean from a given number of stars in space?

D. Content: Operations (Addition, Subtraction, Unknowns)

Let's think back to the original problem: The baker starts with 53 pies, 36 of which are cherry and the rest are apple. Then the baker bakes more apple pies and there are now 42 apple pies. Using that information, what does the box represent in each equation below?

$$36 + \square = 53$$

$$53 - 36 = \square$$

$$42 - 36 = \square$$

$$42 + 36 = \square$$

E. Content: Equality (Unknowns)

Let's look at some other equations with unknowns.

$$3 + 19 = \square$$

$$\square + 6 = 18$$

$$2 + 7 = \square + 4$$

What does the box, or unknown, mean in each equation?

- What does the equal sign mean in each equation? Are the three equal signs different in any way?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

The question is about finding the number of apple pies the baker made compared to how many apple pies the baker started with. It asks how many more apple pies were made.

- Do you have to take time to think about this question? Why do you think that is?

The question has to be thought about because it is not asking for the total number of pies made, which is given as 53. The question is not asking for the number of cherry pies. That is 36. The question is asking how many more apple pies the baker made, which means we need to find how many apple pies the baker had to start.

B. Problem Solving: Strategy

What is the first thing you want to do? Why?

Find what we know. We know the number of cherry pies, the number of pies the baker started with, and the total number of apple pies after the baker made more. Then we can calculate what we don't know, which is the number of apple pies the baker had to start. This will help us find the number of additional apple pies made.

- Could you answer the question in your head, without writing anything down? If so, how would you start?

We could count up from 36 to 53. We could also subtract 36 from 53.

- Could you answer the question with a drawing? If so, how would you start?

A number line, from 0–50, could help. Start with a point at 36 and count the jumps from 36 to 53, which is 17. This names the number of apple pies the baker started with.

- Could you answer the question with objects? If so, how would you start?

Start with 53 objects and move 36 of them to the side to represent the cherry pies. The remaining objects show the number of apple pies the baker started with.

- Could you answer the question with equations? If so, how would you start?

There are many ways to use an equation. One way is to start with the number of cherry pies, 36, and add the unknown for apple pies to equal the total, 53. 36 + ☐ = 53

C. Content: Operations (Addition, Subtraction, Meaning)

Let's talk about adding and subtracting.

When is it OK to add two numbers together? When is it not OK?

It's always ok to add two numbers together if that is what is required in a non-contextual situation. It gets more challenging to make sense of a sum when a context is added.

- Can I add a given quantity of apples to a given quantity of oranges?

Yes, this could give the total number of fruits in a bowl, for example.

- Can I add a given quantity of fish in the ocean to a given quantity of stars in space?

Yes, but it probably wouldn't be very meaningful. This sum could be interpreted as a total number of objects in the universe for this situation of fish and stars. Realistically, though, it does not make much sense to do so.

When is it OK to subtract two numbers? When is it not OK?

It's always OK to subtract two numbers if that is what is required in a non-contextual situation. It gets more challenging to make sense of a difference when a context is added.

- Can I subtract a given quantity of apples from a given quantity of oranges?

  Yes, for example, we could subtract 5 apples -2 oranges to compares the sizes of the fruit collections. In this case, 5 apples minus 2 oranges equals 3 more apples than oranges. However, subtraction as take-away will not make sense in this context.

- Would there ever be a reason to subtract a given number of fish in the ocean from a given number of stars in space?

  It could tell us how many more stars there are there than fish, but that probably isn't very meaningful.

D. Content: Operations (Addition, Subtraction, Unknowns)

Let's think back to the original problem: The baker starts with 53 pies, 36 of which are cherry and the rest are apple. Then the baker bakes more apple pies and there are now 42 apple pies. Using that information, what does the box represent in each equation below?

$$36 + \square = 53$$

$$53 - 36 = \square$$

$$42 - 36 = \square$$

$$42 + 36 = \square$$

In the first two equations, the box represents the number of apple pies the baker started with: 17. In the third equation, the box represents how many more apple pies than cherry pies there are in all. In the fourth equation, the box represents the total number of pies (apple and cherry) after more apple pies are baked.

E. Content: Equality (Unknowns)

Let's look at some other equations with unknowns.

$$3 + 19 = \square$$

$$\square + 6 = 18$$

$$2 + 7 = \square + 4$$

What does the box, or unknown, mean in each equation?

The box represents a value we do not know. In the first equation, the box is equal in value to the sum of 3 and 19. In the second equation, the box is equal in value to the quantity that when added to 6 equals 18. In the last equation, the unknown represents the value that, when increased by 4, is equal to 2 + 7.

- What does the equal sign mean in each equation? Are the three equal signs different in any way?

  In each of the equations the equal sign is a statement that one side is equal in value to the other. Be sure, however that students can flexibly interpret the equal sign as relational. An operational perspective, such as "the result when you add..." will make interpreting the third equation challenging, for example.

2.2 Shareables*

Use the information to answer the question.

A baker has 36 cherry pies and some apple pies. In all, there are 53 pies. The baker then bakes more apple pies. There are now 42 apple pies.

How many more apple pies did the baker bake? Enter the answer in the box.

☐ pies

D.

$$36 + \square = 53$$

$$53 - 36 = \square$$

$$42 - 36 = \square$$

$$42 + 36 = \square$$

E.

$$3 + 19 = \square$$

$$\square + 6 = 18$$

$$2 + 7 = \square + 4$$

2.3 Operations and Place Value

Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (or a bundle of 10), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

ITEM ALIGNMENT

CCSS: 2.NBT.B.6

TX: 2.4.B

This item focuses on understanding and applying place value concepts to complete operations of addition and subtraction. However, it also provides the opportunity to talk about composing and decomposing numbers using properties, and to apply thinking strategies to solve.

THE CONVERSATION STARTER

Add. Move numbers to show the answer.

$$\begin{array}{r} 23 \\ 17 \\ 29 \\ +\,31 \\ \hline \end{array}$$

0 1 2 3 4 5 6 7 8 9

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Strategy

How do you want to solve this problem?

- Can you describe another way to solve the problem?
- Which way would you prefer to solve the problem? Why?
- What are some ways you could group numbers to add them together?

B. Content: Operations (Addition, Meaning)

Let's talk about addition. What does it mean to add?

- Is it ok to add apples and oranges? What is an example?
    - Is it ok to add apples and cars?
- Can I add tens and ones? Why or why not?
- Is it ok to say 5 apples plus 6 oranges is 5 + 6? Why or why not?

C. Content: Place Value (Meaning)

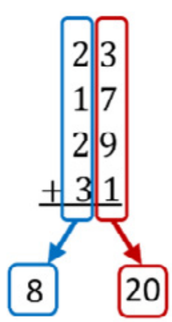
Consider the number 23, the first number in the original problem. What does the 2 mean in 23?

- If 23 represents the number of cookies in a jar, what does the 2 mean?
- Using addition, how else could you represent 23?

When adding two or more 2-digit numbers, do you have to work from left to right to find the sum? Is it possible to work from right to left? Explain.

D. Content: Place Value (Meaning)

Tell me when you see how this student's work *almost* shows the answer.



CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Strategy

How do you want to solve this problem?

Answers will vary. Do students use a procedure or some other strategy?

- Can you describe another way to solve the problem?

  Answers will vary. This is a great opportunity to listen to how students are thinking about place value and operations.

- Which way would you prefer to solve the problem? Why?

- What are some ways you could group numbers to add them together?

  Some options are to combine 3 + 7 and 9 + 1 to make 10. Students may also wish to decompose each number into expanded form as follows:

  - ***20 + 3***
  - ***10 + 7***
  - ***20 + 9***
  - ***30 + 1***

  Students may wish to use partial addition by first adding the ones — 3 + 7 + 9 + 1 = 20, and then adding the tens, 20 + 10 + 20 + 30 = 80, and then adding the results together: 20 + 80 = 100. Or, some students may wish to first add the tens then add the ones.

B. Content: Operations (Addition, Meaning)

Let's talk about addition. What does it mean to add?

To put amounts together to make a total.

- Is it OK to add apples and oranges? What is an example?

  Yes, if we want to find the total number of fruits in a bowl.

  - Is it OK to add apples and cars?

    It can be OK, but it may be strange to do so. If someone saw a parking lot that had only cars and apples in it and wondered how many total things were on the lot, they could add the apples and cars.

- Can I add tens and ones? Why or why not?

  Not directly, but you can think about the tens as a number of ones and then add. To add 2 tens and 3 ones you can think: 2 tens is 20 ones, and 20 ones plus 3 ones is 23 ones.

- Is it OK to say 5 apples plus 6 oranges is 5 + 6? Why or why not?

  5 tens have a value of 50, not 5. So, while the number is 5 in 56, the value is 50. So, 5 tens plus 6 ones, is 50 + 6.

C. Content: Place Value (Meaning)

Consider the number 23, the first number in the original problem. What does the 2 mean in 23?

The 2 in 23 means 2 tens, or 10 + 10, or 20.

- If 23 represents the number of cookies in a jar, what does the 2 mean?

  If there are 23 cookies in a jar, then the 2 means 2 groups of 10 cookies, or 10 + 10, or 20 cookies.

- Using addition, how else could you represent 23?

  Students should understand that a number can be decomposed in many ways. For example:

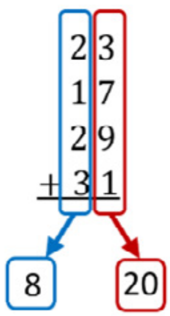
  - ***10 + 10 + 3***
  - ***5 + 5 + 5 + 5 + 3***
  - ***19 + 4***
  - ***1 + 22***

When adding two or more 2-digit numbers, do you have to work from left to right to find the sum? Is it possible to work from right to left? Explain.

It is not necessary to work in either direction. When focused on place value, using this current example, we can add 20 + 10 + 20 + 30 = 80 and 3 + 7 + 9 + 1 = 20, and then the total is 80 + 20 = 100.

D. Content: Place Value (Meaning)

Tell me when you see how this student's work *almost* shows the answer.



The student correctly added the ones because 3 + 7 = 10, 9 + 1 = 10, and 10 + 20 = 20. The student added the digits in the tens place correctly as well. 2 + 1 + 2 + 3 = 8. Next, the student needs to understand that the 8 represents 80.

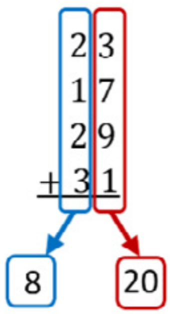
2.3 Shareables*

Add. Move numbers to show the answer.

```
  2 3
  1 7
  2 9
+ 3 1
-----
```

0 1 2 3 4 5 6 7 8 9

D.



2.4 Operations and Place Value

Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (or a bundle of 10), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

ITEM ALIGNMENT

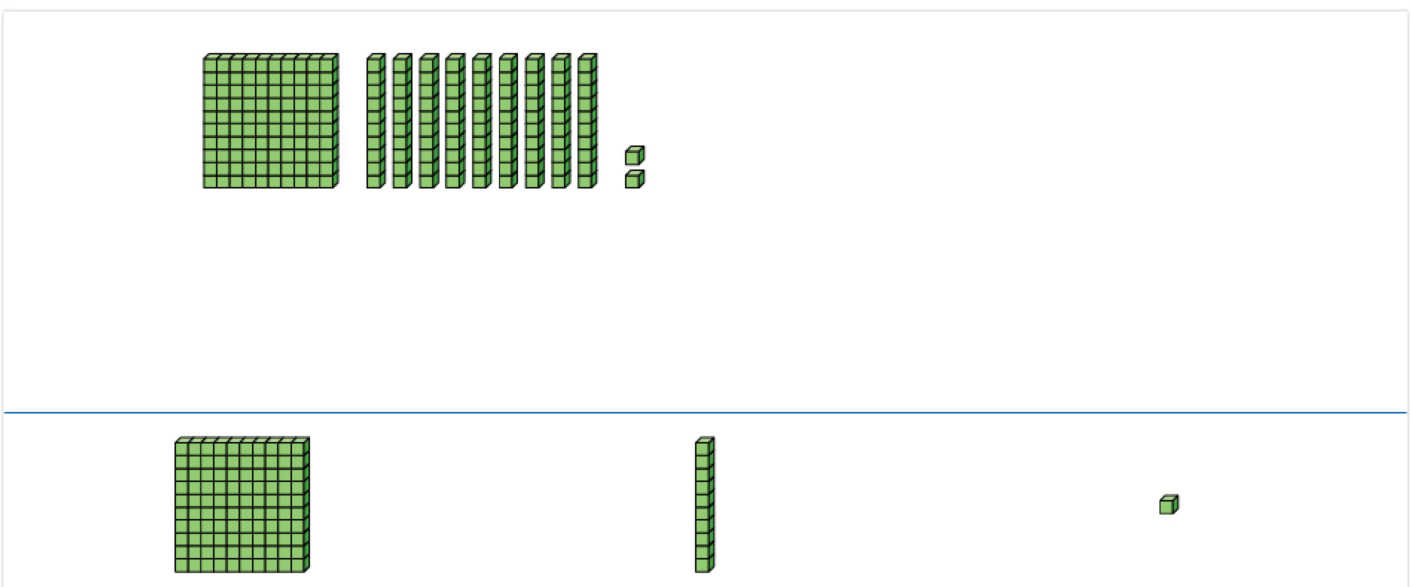
CCSS: 2.NBT.B.7

This item focuses on understanding place value concepts, comparisons, and representation. However, it also provides the opportunity to work with concrete models and base ten.

THE CONVERSATION STARTER

Use base-ten blocks to show the answer.

Add some more base-ten blocks to the group to show a total of 305 using the fewest number of blocks.



CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

What do you need to know to answer this question?

- What number is represented in the picture?
- In your own words, what is the question asking?

B. Content: Place Value (Meaning)

The number represented in the problem is 192. How else could you show 192 using blocks?

- Could you represent 192 with fewer blocks? Why or why not?
- How could you show 192 if you didn't have any hundreds blocks?
- How could you show 192 using an equation?

C. Content: Place Value (Meaning)

How many tens rods could be used to show 740? How many can be used to show 745?

- If I didn't have any hundreds flats, how many tens rods would it take to show 740? How many to show 745?
- Let's say a baker baked 375 cookies and put them into bags of 10. How many bags of 10 should there be? Why?

D. Content: Operations and Equations

The original problem asks us to "add some more" blocks to show 305.

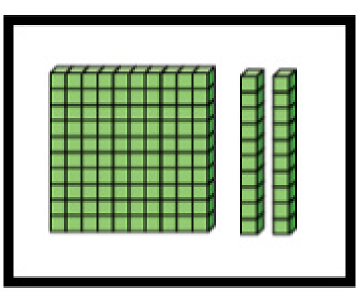
How could you write this situation as an equation using addition?

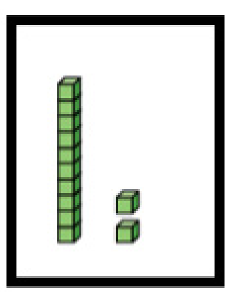
- Could you write this situation as an equation using subtraction?

Let's talk about addition. What does it mean to add?

- Is it OK to add apples and oranges? What is an example?
  - Is it OK to add apples and cars?
- Can I add tens and ones? Why or why not?
- Is it OK to say 5 apples plus 6 oranges is 5 + 6? Why or why not?
- Is it OK to say 5 tens plus 6 ones is 5 + 6? Why or why not?

How are these images alike? How are they different?





- Could these two images represent the same number?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

What do you need to know to answer this question?

We need to know what each block represents. The largest flat block represents 100. The rod represents 10 and the single blocks represent 1. So, 1 flat (100) plus 1 rod (10) plus 1 single block (1) equals 111.

- What number is represented in the picture?

  192; The blocks show 1 hundreds flat, or 100; 9 tens rods, or 90; and 2 single blocks, or 2. 100 + 90 + 2 = 192.

- In your own words, what is the question asking?

  Some possible responses are:

  - ***How many blocks do I need to add to go from 192 to 305?***
  - ***If I show 305 in blocks, how will it look different from 192?***
  - ***How many hundreds, tens, and ones do I need to add to 192 to equal 305?***

B. Content: Place Value (Meaning)

The number represented in the problem is 192. How else could you show 192 using blocks?

There are many possibilities. For example, one flat plus 80 rods plus 12 single blocks. Also, 19 rods and 2 single blocks show 192.

- Could you represent 192 with fewer blocks? Why or why not?

  No, because there are not enough ones (10) to trade for a tens rod. There are also not enough tens rods (10) to trade for a hundreds flat.

- How could you show 192 if you didn't have any hundreds blocks?

  Some possible responses are:

  - ***19 groups of tens rods and 2 ones = 19 tens = 100 + 90. 190 + 2 = 192***
  - ***18 groups of ten rods and 12 ones = 18 tens = 100 + 80. 180 + 12 = 192***

- How could you show 192 using an equation?

  Some possible responses are:

  - ***190 + 2 = 192***
  - ***180 + 12 = 192***
  - ***100 + 90 + 2 = 192***
  - ***50 + 50 + 90 + 2 = 192***

C. Content: Place Value (Meaning)

How many tens rods could be used to show 740? How many can be used to show 745?

If we were to use the fewest number of blocks to represent the 740 (or 745), then there would be 4 tens rods along with 7 hundreds flats – plus 5 ones blocks to show 745. However, we could also represent 740 with 74 tens rods.

- If I didn't have any hundreds flats, how many tens rods would it take to show 740? How many to show 745?

  If there were no hundreds flats, 700 could be shown with tens rods by showing 7 groups of 10 tens rods and then showing 40 with 4 more tens rods. To show 745, just add 5 ones blocks.

- Let's say a baker baked 375 cookies and put them into bags of 10. How many bags of 10 should there be? Why?

  If there were 375 cookies put into bags of 10 cookies, it would take 37 bags of 10 cookies, and 1 bag of 5 cookies, or 38 bags total. This is because there are 37 tens in 375, and 5 ones.

D. Content: Operations and Equations

The original problem asks us to "add some more" blocks to show 305.

How could you write this situation as an equation using addition?

Some possible responses are:

- ***192 + ☐ = 305***
- ***☐ + 192 = 305***
- ***192 + ☐ + ☐ + ☐ = 305***

  Here, a student may connect the equation to the idea of additional flats, rods, and ones. For example, 192 + 100 + 10 + 1 = 305.

- Could you write this situation as an equation using subtraction?

  Some possible responses are:

  - ***305 − 192 = ☐***
  - ***305 − ☐ = 192***
  - ***192 + ☐ + ☐ + ☐ = 305***

    Here, a student may connect the equation to the idea of additional flats, rods, and ones. For example, 192 + 100 + 10 + 1 = 305.

Let's talk about addition. What does it mean to add?

To put amounts together to make a total.

- Is it OK to add apples and oranges? What is an example?

  Yes, if we want to find the total number of fruits in a bowl.

  - Is it OK to add apples and cars?

    It can be OK, but it may be strange to do so. If someone saw a parking lot that had only cars and apples in it and wondered how many total things were on the lot, they could add the apples and cars.

- Can I add tens and ones? Why or why not?

  Not directly, but you can think about the tens as a number of ones and then add. To add 2 tens and 3 ones, you can think: 2 tens is 20 ones, and 20 ones plus 3 ones is 23 ones.

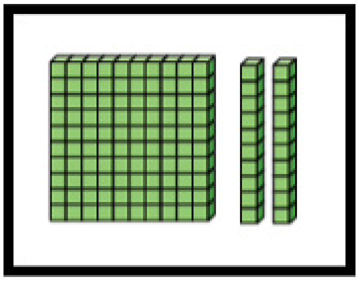
- Is it OK to say 5 apples plus 6 oranges is 5 + 6? Why or why not?

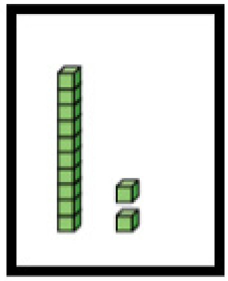
  Apples and oranges can be added to find the total number of fruits. However, it would not make sense to add 5 apples and 6 oranges together to tell how many total apples you have.

- Is it OK to say 5 tens plus 6 ones is 5 + 6? Why or why not?

  5 tens have a value of 50, not 5. So, while the number is 5 in 56, the value is 50. So, 5 tens plus 6 ones is 50 + 6.

How are these images alike? How are they different?





They are alike in that they use the same digits in the same order, a 1 followed by a 2. They are different in that they show two different values—120 and 12.

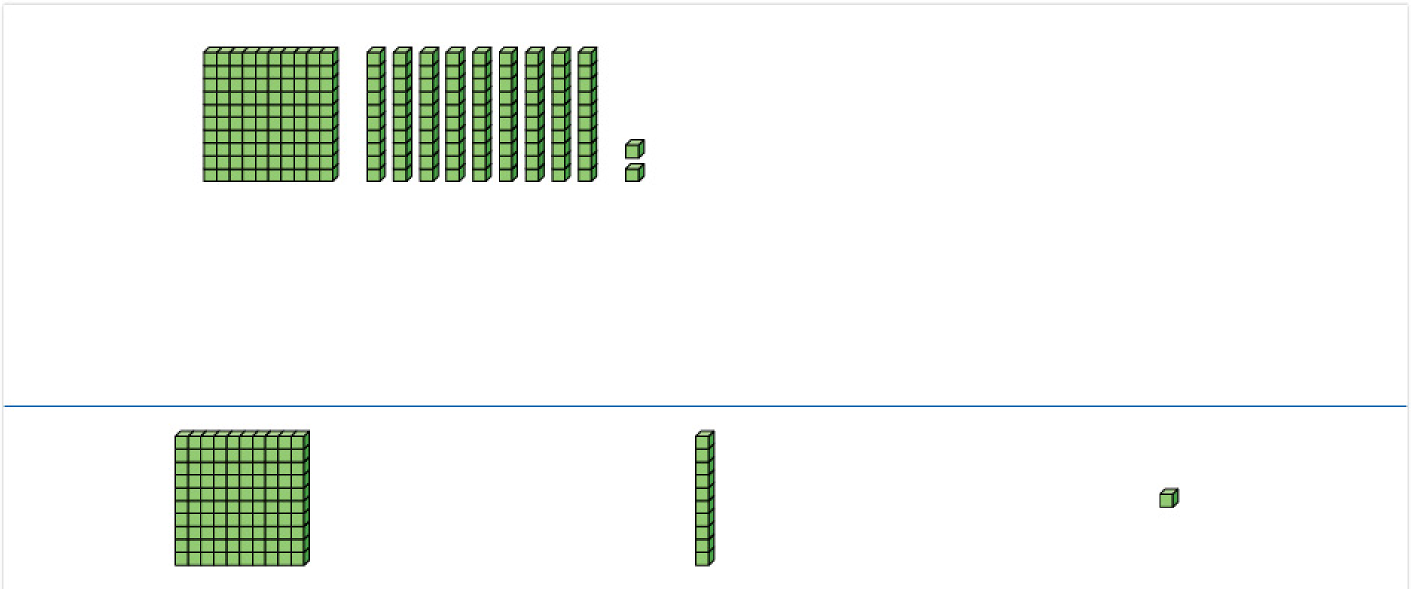
- Could these two images represent the same number?

  Yes, if the cubes in each image represent different quantities. For example, the first image shows 120 if each unit cube represents 1. The second image also shows 120 if each unit cube represents 10.

2.4 Shareables*

Use base-ten blocks to show the answer.

Add some more base-ten blocks to the group to show a total of 305 using the fewest number of blocks.



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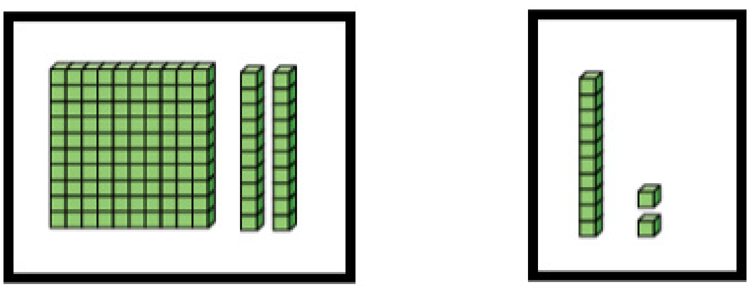


2.5 Comparisons and Measurement

Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is 3 times as large as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is 4 meters more than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

ITEM ALIGNMENT

CCSS: 2.MD.A.5

NE: 2.3.3.D

TX: 2.9.B

VA: 2.9.E

This item focuses on integrating measurement and operations. However, it also provides the opportunity to apply operations in a real-world measurement problem scenario.

THE CONVERSATION STARTER

Use the information to answer the question.

Kelsey is 13 inches taller than her sister. Kelsey is 53 inches tall.

How tall is Kelsey's sister? Enter the answer in the box.

[ ] inches

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

- Who is taller, Kelsey or her sister? How do you know?

B. Problem Solving: Strategy

To answer this question, what is the first thing you want to do? Why?

- Could you answer the question in your head, without writing anything down? If so, how would you start?
- Could you answer the question with a drawing? If so, how would you start?
- Could you answer the question with objects? If so, how would you start?
- Could you answer the question with equations? If so, how would you start?
- Could you write an equation that uses addition? Subtraction?

C. Content: Measurement (Meaning)

What does it mean to measure something?

- What does it mean to measure length?
- If a student says something like "it means to find out how many inches (or feet or ?) are in the length," follow up with:
  - Can you measure length with a piece of string, a pencil, or a hand?
- What does it mean to measure the weight of something?

D. Content: Measurement (Inches and Centimeters)

Can you estimate an inch without a ruler?

- Use your thumb and index finger to show me one inch and one centimeter.
- About how long is 13 inches?

E. Content: Operations (Subtraction)

Let's talk about subtraction.

- When is subtraction helpful?
- When is it OK to subtract two numbers? When is it not OK?
- Can I subtract a given quantity of apples from a given quantity of oranges?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

Listen for an understanding about how the given numbers relate to one another.

- Who is taller, Kelsey or her sister? How do you know?

  Kelsey is taller than her sister. The problems states, "Kelsey is 13 inches taller than her sister."

B. Problem Solving: Strategy

To answer this question, what is the first thing you want to do? Why?

Answers will vary. A student may want to draw a picture or write an equation.

- Could you answer the question in your head, without writing anything down? If so, how would you start?

  Think about what is known—Kelsey is 53 inches tall. Then think about how her height compares to her sister's height.

- Could you answer the question with a drawing? If so, how would you start?

  You could draw vertical line segments to show the relationship between the two heights.

- Could you answer the question with objects? If so, how would you start?

  Objects, such as tens rods and ones blocks could be used to represent the 53 inches and tens rods and ones blocks to represent the13 inches. The rods and blocks could be added to find Kelsey's sister's height.

- Could you answer the question with equations? If so, how would you start?

  For example, a student may write 53 + 13 = ☐.

- Could you write an equation that uses addition? Subtraction?

  For example, a student may write 53 + 13 = ☐ or ☐ – 13 = 53.

C. Content: Measurement (Meaning)

What does it mean to measure something?

It means to find how many copies of one thing it takes to match another thing. The number of copies is the measure.

When measuring something it is important to be clear about which attribute is being measured. Is it length? Is it area? An object that has the same attribute can be used as a standard unit of measure. Then we can compare multiplicatively—how many copies of the standard unit does it take to match the attribute of the object being measured?

- What does it mean to measure length?

  It means to find how many copies of one length—a distance—it takes to match another length.

- If a student says something like "it means to find out how many inches (or feet or ?) are in the length," follow up with:

  - Can you measure length with a piece of string, a pencil, or a hand?

    Yes. For example, "A desk is 4 pencils wide." "A door is 25 hands tall."

- What does it mean to measure the weight of something?

  If a pet weighs 35 pounds it means that there is a standard weight of one pound, and the pet's weight is 35 copies of that standard weight.

D. Content: Measurement (Inches and Centimeters)

Can you estimate an inch without a ruler?

Yes.

- Use your thumb and index finger to show me one inch and one centimeter.

  Listen or watch for how the student estimates one inch and one centimeter.

- About how long is 13 inches?

  Listen or watch for how the student estimates 13 inches.

E. Content: Operations (Subtraction)

Let's talk about subtraction.

- When is subtraction helpful?

  Subtraction is helpful to compare two numbers to find the difference between them.

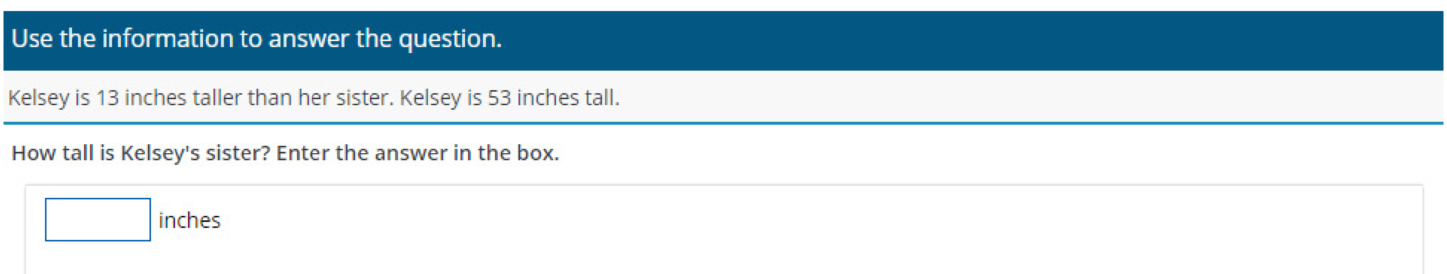
- When is it OK to subtract two numbers? When is it not OK?

  It's always OK to subtract two numbers if that is what is required in a non-contextual situation. It gets more challenging to make sense of a difference when a context is added.

- Can I subtract a given quantity of apples from a given quantity of oranges?

  Yes, for example, we could subtract 5 apples – 2 oranges to compares the sizes of the fruit collections. In this case, 5 apples minus 2 oranges equals 3 more apples than oranges. However, subtraction as take-away will not make sense in this context.

2.5 Shareables*


Use the information to answer the question.

Kelsey is 13 inches taller than her sister. Kelsey is 53 inches tall.

How tall is Kelsey's sister? Enter the answer in the box.

[ ] inches

2.6 Measurement

Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

ITEM ALIGNMENT

CCSS: 2.MD.B.5

TX: 2.9.E

This item focuses on measurement of length. However, it also provides an opportunity to talk about measurement in general, which is foundational to measuring length, and to examine flexibility in measurement and the comparison of different units.

THE CONVERSATION STARTER

This question has two parts. Use the ruler tool to measure the eraser in both centimeters (cm) and inches. Then answer Part A and Part B.

Part A

How long is the eraser to the nearest centimeter and to the nearest inch? Enter the answers in the boxes.

Length in centimeters: ______

Length in inches: ______

Part B

Why are the number of centimeters and the number of inches different? Select one choice from each set to complete the sentence.

The number of centimeters is **[greater / less]** than the number of inches because one centimeter is **[longer / shorter]** than one inch.

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

What do you need to answer this question?

- Why do you need a ruler? Could you answer the question without a ruler?
- Can we estimate an inch without a ruler?
  - Use your thumb and index finger to show me one inch and one centimeter.
- What do you think the answers to Part A are?

B. Content: Measurement (Meaning)

What does it mean to measure something?

- What does it mean to measure length?
- If a student says something like "it means to find out how many inches (or feet or ?) are in the length," follow up with:
  - Can you measure length with a piece of string, a pencil, or a hand?
- What does it mean to measure the weight of something?

C. Content: Measurement (Length)

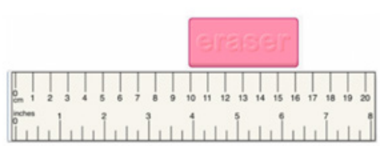
What else could we use to measure the length?

Can you name an object that has length?

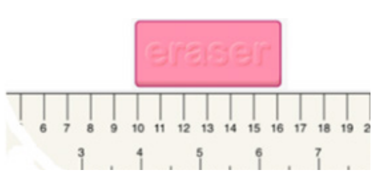
- How can you use that object to measure length?
- Why do we use measurement units such as inches or centimeters?

D. Content: Measurement

Suppose someone tries to measure the length of the eraser like this. What would you tell them?



- Tell me when you see that the eraser in the diagram is as long as 6 copies of one centimeter.
- Tell me when you can see that the eraser is about 2 copies of one inch.
- If a student is firm on the need to start at zero, hand them a paper ruler with the left end cut and ask:
  - Can you measure the eraser with this ruler?



  - Can you explain what it means to measure the length of this eraser?

E. Extension: Measurement

The eraser is 6 centimeters long. It's also around 2 inches long. Is it longer if we use centimeters?

- Why is the length of the eraser measured in centimeters a greater number than the length of the eraser measured in inches?
- Could we measure the length of the eraser with this blue line?

- What might be the measure in "blue lines"?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

What do you need to answer this question?

- Why do you need a ruler? Could you answer the question without a ruler?

  Rulers quickly provide a standard inch that will be the same for everyone. You could use something else that allows you to compare to inches, such as a piece of string that is 1 inch long.

- Can we estimate an inch without a ruler?

  Yes.

  - Use your thumb and index finger to show me one inch and one centimeter.

    Listen or watch for how the student estimates one inch and one centimeter.

- What do you think the answers to Part A are?

  Listen or watch for how the student estimates the length of the eraser.

B. Content: Measurement (Meaning)

What does it mean to measure something?

It means to find how many copies of one thing it takes to match another thing. The number of copies is the measure.

When measuring something, it is important to be clear about which attribute is being measured. Is it length? Is it area? An object that has the same attribute can be used as a standard unit of measure. Then we can compare multiplicatively—how many copies of the standard unit does it take to match the attribute of the object being measured?

- What does it mean to measure length?

  It means to find how many copies of one length—a distance—it takes to match another length.

- If a student says something like "it means to find out how many inches (or feet or ?) are in the length," follow up with:

  - Can you measure length with a piece of string, a pencil, or a hand?

    Yes. For example, "A desk is 4 pencils wide." "A door is 25 hands tall."

- What does it mean to measure the weight of something?

  If a pet weighs 35 pounds it means that there is a standard weight of one pound, and the pet's weight is 35 copies of that standard weight.

C. Measurement (Length)

What else could we use to measure the length?

Any object that has length.

Can you name an object that has length?

For example: a pencil, a marker, a hand.

- How can you use that object to measure length?

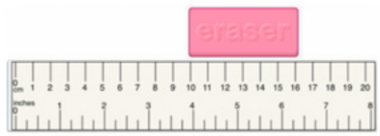
  It can be used to measure length in terms of the object. For example, "A desk is 4 pencils wide." "A door is 25 hands tall."

- Why do we use measurement units such as inches or centimeters?

  To be consistent, we need to have at least one shared length we all agree on. When we use fingers, for example, not everyone's fingers are all the same length.

D. Content: Measurement

Suppose someone tries to measure the length of the eraser like this. What would you tell them?



Listen for students' understanding of measurement. Some students may say "move the eraser to start at zero." Some students may say it's OK to measure this way.

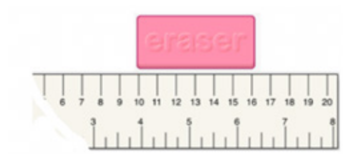
- Tell me when you see that the eraser in the diagram is as long as 6 copies of one centimeter.

  A student may see that there are 6 lengths of one centimeter in the length of the eraser. Or a student may see 16 – 10 = 6.

- Tell me when you can see that the eraser is about 2 copies of one inch.

  A student may say that there are 2 lengths of one inch in the length of the eraser. A student may see 6 – 4 = 2. It is also common for students to respond with, "first I would need to move the eraser to start at zero."

- If a student is firm on the need to start at zero, hand them a paper ruler with the left end cut and ask:
  - Can you measure the eraser with this ruler?



  - Can you explain what it means to measure the length of this eraser?

    We are comparing the length of the eraser to the length of 1 centimeter or 1 inch to see how many copies of 1 centimeter or 1 inch long the length of the eraser is.

Here are some other ideas to explore. These can further push the idea of what it means to measure:

- How might a student be thinking if they indicate that the length of the eraser is 16 cm? How might their thinking be adjusted? (The student may be looking at the end of the eraser, which is at the 16 cm mark.) To adjust this thinking, a student might need to return to the meaning of measurement focusing on determining the number of copies of a 1-cm length needed to match the length of the eraser.
- How might a student be thinking if they indicate that the length of the eraser is 7 cm? (Sometimes students count the marks. In this case, if we count the marks for 10, 11, 12, 13, 14, 15, 16...there are 7 marks.) To adjust this thinking, the student should be guided to count 1-centimeter lengths, or gaps, rather than the marks themselves. The student should be thinking about the attribute of length and comparing multiplicatively.

E. Extension: Measurement

The eraser is 6 centimeters long. It's also around 2 inches long. Is it longer if we use centimeters?

No, the eraser is the same length no matter what units we use to measure its length.

- Why is the length of the eraser measured in centimeters a greater number than the length of the eraser measured in inches?

  1 centimeter is smaller than 1 inch, so it takes more centimeter copies to match the length of the eraser.

- Could we measure the length of the eraser with this blue line?

Yes, it has a length.

- What might be the measure in "blue lines"?

It's drawn to be one-half of a blue line, so the length of the eraser might be one-half of a blue line.

2.6 Shareables*

This question has two parts. Use the ruler tool to measure the eraser in both centimeters (cm) and inches. Then answer Part A and Part B.

Part A

How long is the eraser to the nearest centimeter and to the nearest inch? Enter the answers in the boxes.

Length in centimeters: ______

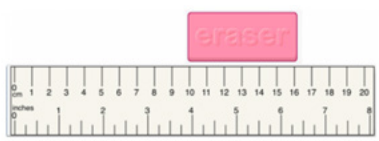
Length in inches: ______

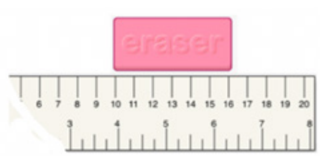
Part B

Why are the number of centimeters and the number of inches different? Select one choice from each set to complete the sentence.

The number of centimeters is **[greater / less]** than the number of inches because one centimeter is **[longer / shorter]** than one inch.

D.





E.

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